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NOTE UPON A REPRESENTATION IN SPACE OF THE ELLIPSES DRAWN BY AN ELLIPSOGRAPH.*

By Prof. E. M. Blake, Greencastle, Ind.

In a fixed plane P let there be two straight lines OX and OY mutually perpendicular. In another plane P_1 let there be two fixed points A and B distant $2a$ from each other, and let C, the middle point of the line joining A, B, be the pole, and CA the initial line of a system of polar coordinates for the determination of the position of any point in P_1. Let ρ and θ be the radius vector and vectorial angle, respectively, of a point p.

If now the points A and B be required to move along OX and OY, respectively, the point p of the plane P_1 will describe an ellipse upon the plane P. The fact is well known and is often used in mechanisms for drawing ellipses and producing elliptical motion. To these belongs the trammel. The equation of the ellipse is

$$(a^2 + \rho^2 - 2a\rho \cos \theta)\, x^2 + (a^2 + \rho^2 + 2a\rho \cos \theta)\, y^2 \\ - 4a\rho \sin \theta \,.\, xy = (\rho^2 - a^2)^2 \quad (1)$$

referred to OX and OY as coordinate axes. It is proposed to give a simple space representation for the two-fold infinity of ellipses obtained by using all points of P_1 for tracing points, *i. e.* by permitting ρ and θ, in equation (1), to assume any values.

By rotating the axes OX and OY through the angle $\theta 22$ in a positive direction, keeping the origin fixed, the equation (1) becomes

$$\frac{x^2}{(\rho + a)^2} + \frac{y^2}{(\rho - a)^2} = 1 \,. \qquad (2)$$

This represents an ellipse referred to its centre and axes. Since it is independent of θ, every point of the circumference of a circle of radius ρ and centre C in P_1 generates an ellipse congruent and concentric to (2), and all points of a half line from O to infinity in the plane P_1 generate ellipses having the same centre O and axes in the same directions. Hence, except for the directions of their axes (2) represents all ellipses drawn by the ellipsograph.

Let us now erect at O an axis OZ perpendicular to P, and at the height ρ above P pass a plane parallel to it. In this second plane draw the ellipse

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* Read before the Chicago Section of the American Mathematical Society, April 24th, 1897.

(2) with its axes parallel to OX and OY. Let this be done for every value of ρ, then the ellipses form the surface

$$\frac{x^2}{(z+a)^2}+\frac{y^2}{(z-a)^2}=1\,. \tag{3}$$

Permitting z to take all values from $-\infty$ to $+\infty$, this surface represents the ellipses generated by points of the line A, B. If the surfaces is rotated in a positive direction through the angle $\theta/2$, it represents the ellipses generated by points of the line through C, making the angle θ with CA. This is the required space representation for the ellipses.

The sections of the surface (3), by the planes $z=0$, $z=\pm\infty$, are circles which are generated respectively by C and the infinitely distant points of the line AB. The straight line joining $(2a, 0, a)$, $(-2a, 0, a)$, and that joining $(0, 2a, -a)$, $(0, -2a, -a)$ are nodal lines, and are the ellipses generated by the points A and B, respectively. The four extremities $(2a, 0, a)$, $(-2a, 0, a)$, $(0, 2a, -a)$, $(0, -2a, -a)$, are pinch points.

The surface is a ruled surface generated by a straight line which touches two fixed lines at right angles in parallel planes (the nodal lines of the surface), and whose segment between the fixed lines has the length $\sqrt{2}$ times the distance between the parallel planes.*

From this it follows that the generating line makes constantly an angle of 45 with the planes of the nodal lines, and that a point on the generatrix, at a fixed distance from a nodal line, describes a plane curve in a plane parallel to the planes of the nodal lines. The curve is consequently an ellipse, which establishes a close analogy between the plane ellipsograph and this one in space. If we do not condition the segment of the generatrix of the surface to bear a fixed relationship to the distance between its nodal lines, the former is what the latter becomes with its nodal lines brought into the same plane.

PURDUE UNIVERSITY, *April* 20, 1897.

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* For this theorem I am indebted to Prof. C. A. Waldo, Purdue University.